Polarization

- Linear, Circular, in general elliptical

$$\tilde{E}(z) = \hat{x}\tilde{E}_x(z) + \hat{y}\tilde{E}_y(z)$$

$$\tilde{E}_x(z) = E_{x0}e^{-jkz}, \quad \tilde{E}_y(z) = E_{y0}e^{-jkz}$$

$$E_{x0} = a_x, \quad E_{y0} = a_y e^{j\delta}$$

$$\tilde{E}(z) = (\hat{x}a_x + \hat{y}a_y e^{j\delta})e^{-jkz}$$

δ is phase difference

instantaneous field

$$E(z,t) = \mathrm{Re}\left[\tilde{E}(z)e^{j\omega t}\right]$$

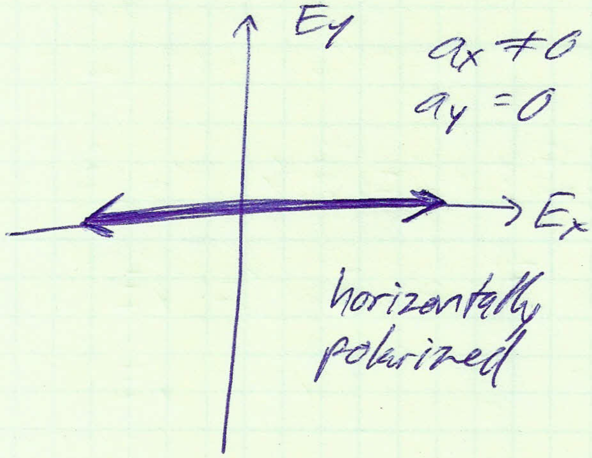
magnitude

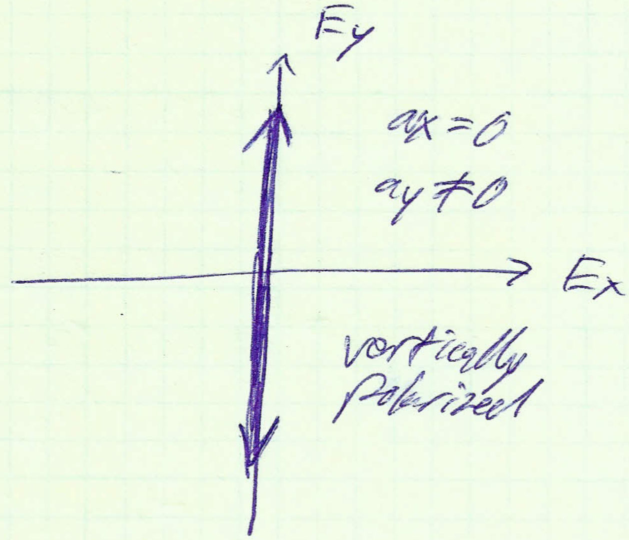
$$|E(z,t)| = \left[E_x^2(z,t) + E_y^2(z,t)\right]^{1/2}$$

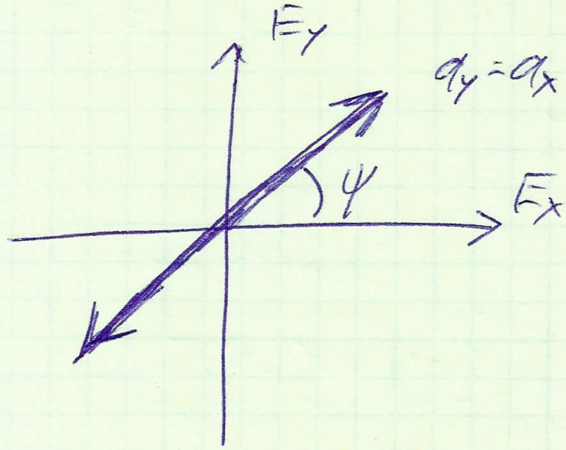
inclination angle

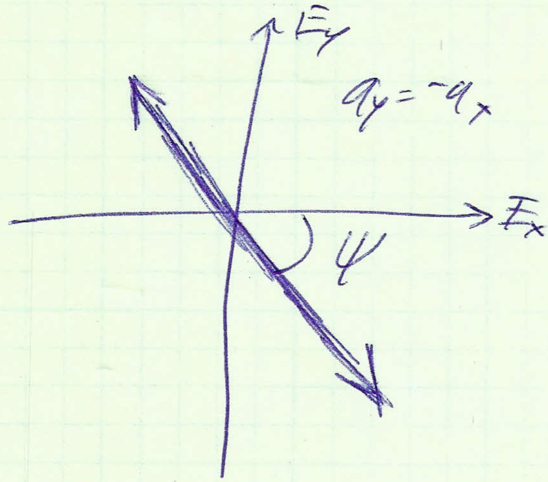
$$\psi(z,t) = \tan^{-1}\left[\frac{E_y(z,t)}{E_x(z,t)}\right]$$

Linear Polarization









Circular Polarization

$$\tilde{E}(z) = \left(\hat{x}a + \hat{y}a e^{j\pi/2}\right) e^{-jkz} = a(\hat{x} + j\hat{y}) e^{-jkz}$$

Left-hand circular polarization

$$E(z,t) = \mathrm{Re}\left[\tilde{E}(z) e^{j\omega t}\right]$$

$$= \hat{x} a \cos(\omega t - kz) + \hat{y} a \cos(\omega t - kz + \pi/2)$$

$$= \hat{x} a \cos(\omega t - kz) - \hat{y} a \sin(\omega t - kz)$$

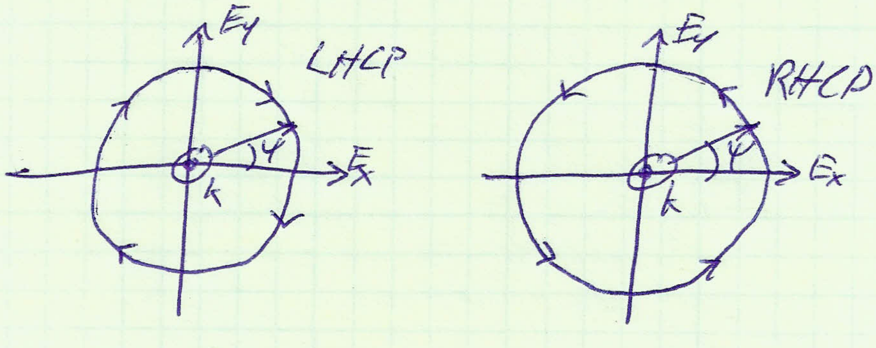
$$|E(z,t)| = a$$

$$\psi(z,t) = \tan^{-1}\left[\frac{E_y(z,t)}{E_x(z,t)}\right] = \tan^{-1}\left[\frac{-a \sin(\omega t - kz)}{a \cos(\omega t - kz)}\right]$$

$$= -(\omega t - kz)$$

→ Inclination angle rotates as wave propagates

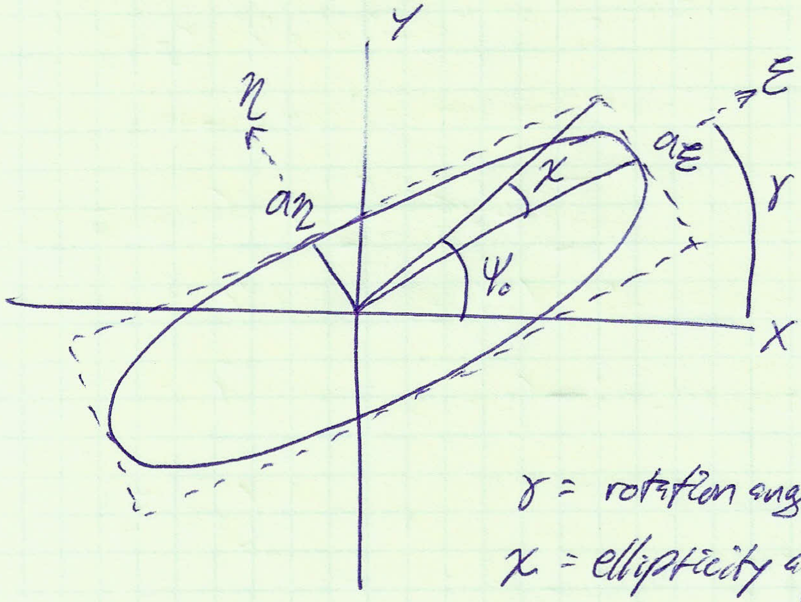
Also rotates in time for a fixed point in space



Definition: fingers of left hand wrap in direction of rotation with time for a fixed point in space. Thumb points in direction of propagation.

This is opposite to direction of rotation ~~of~~ as a function of distance for a fixed time

Elliptical polarization



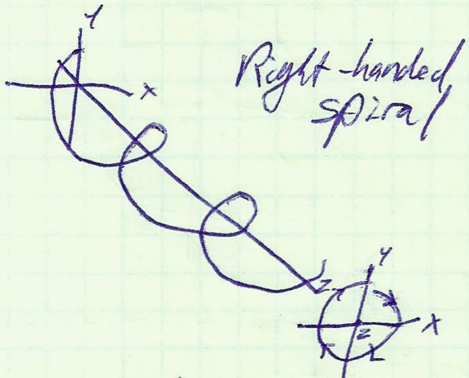
γ = rotation angle

χ = ellipticity angle $= \tan^{-1}\left(\frac{a_\eta}{a_\xi}\right)$

$\frac{a_\xi}{a_\eta}$ = axial ratio

ψ_0 = auxiliary angle $= \tan^{-1}\left(\frac{a_x}{a_y}\right)$

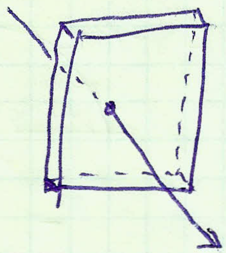
Example: Spiral antenna



Right-handed spiral

left-handed polarization

Example: quarter-wave plate



$n_x \neq n_y$ or $\epsilon_x \neq \epsilon_y$

$k = \omega\sqrt{\mu\epsilon}$

$(\omega\sqrt{\mu\epsilon_x} - \omega\sqrt{\mu\epsilon_y})t = \frac{\pi}{2}$

one component delayed by $\lambda/4$ with respect to the other

can transform linear to circular pol